Solución (1)

1er Examen Parcial (30 pts)
Transferencia de Calor I (TF-2251)

Problema 1

Datos

$r_0 = 10\times10^{-3}\,m.$

$r_1 = 12\times10^{-3}\,m$

$r_2 = 17\times10^{-3}\,m$

$L_f = r_3 - r_2 = 23\times10^{-3}\,m$

$N_f = 12.$

$K_f = 175\ w/mK$

$t_f = 4\times10^{-3}\,m$

$K_0 = 1{,}5\ w/mK$ (varilla)

$K_1 = 0{,}5\ w/mK$ (manga)

$T_s = T_\infty = 25°C$

$h = 20\,w/m^2K$

$\dot{q}_v = 1{,}23\times10^6\,w/m^3$

Se pide

$T(r=0) = ?$

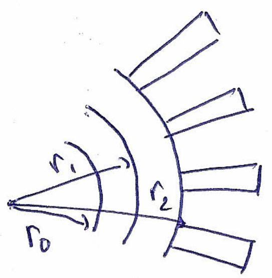
$T(r=r_0) = ?$

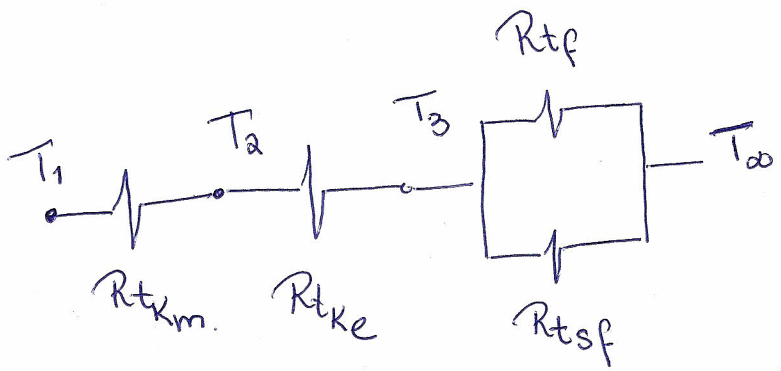
$T(r=r_1) = ?$

$T(r=r_2) = ?$

m: manga aislante
e: estrella.
v: varilla.

Analogía eléctrica





$$Rt_{Km} = \frac{\ln(r_1/r_0)}{2\pi K_m L} = \frac{\ln(12/10)}{2\pi(0{,}5\,w/mK)\,(1m)} = 0{,}058035\ K/w$$

$$Rt_{Ke} = \frac{\ln(r_2/r_1)}{2\pi K_e L} = \frac{\ln(17/12)}{2\pi(175\,w/mK)(1m)} = 3{,}1677\times10^{-4}\ K/w$$

$$R_{tsf} = \frac{1}{h A_{sf}} \qquad q_{sf} = h A_{sf} (T_b - T_\infty) \Rightarrow R_{tsf} = \frac{1}{h A_{sf}}$$

$$A_{sf} = A_b - 12 t_f L = 2\pi r_2 L - 12 t_f L = 2\pi (17 \times 10^{-3} m)(1m) - 12 (4 \times 10^{-3} m)(1m)$$

$$A_{sf} = 5{,}8814 \times 10^{-2} m^2$$

$$R_{tsf} = \frac{1}{200 W/m^2K \times 5{,}8814 \times 10^{-2} m^2} = 8{,}5014 \times 10^{-1} K/W$$

Para las aletas

$q_f = \eta_f A_f h (T_b - T_\infty)$ esto es por definición de eficiencia $\eta_f = \frac{q_f}{h A_f \theta_b}$ (1 aleta)

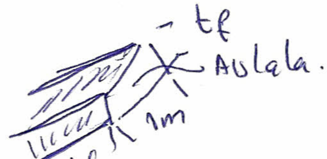
$\Rightarrow q_{aletas} = N_f q_f$

$\Rightarrow q_{aletas} = N_f \eta_f A_f h (T_b - T_\infty)$

$\Rightarrow R_{tf} = \frac{1}{N_f \eta_f A_f h}$

⚠ Condición de borde de la aleta (adiabática en la superficie)

$$A_f = P \times L_f = (2(1m) + 2(4 \times 10^{-3} m))(23 \times 10^{-3} m) =$$



$$A_f = 4{,}618 \times 10^{-2} m^2$$

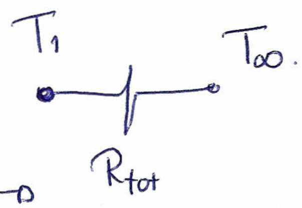
$$\eta_f = \frac{\tanh(m L_f)}{m L_f} \qquad m = \sqrt{\frac{hP}{K_f A_c}} \qquad A_c = 1(m) \times 4 \times 10^{-3} m = 4 \times 10^{-3} m^2$$

$$m = 7{,}5744 \qquad \eta_f = 0{,}99$$

$$R_{tf} = 9{,}113 \times 10^{-2} K/W$$

La Resistencia total equivalente

$$R_{TOT} = R_{tKm} + R_{tKe} + \left(\frac{1}{R_{tf}} + \frac{1}{R_{tsf}}\right)^{-1} = 1,4066\times10^{-1}\ K/W$$



$$q = \frac{T_1 - T_\infty}{R_{TOT}}$$

$$q = \dot{q}\, V_{varilla} = 1,23\times10^{6}\ W/m^3 \left(\pi r_0^2 \cdot L\right) m^3$$

$$q = 386,42\ W$$

$$T_1 = q R_{TOT} + T_\infty$$

$$T_1 = 79,35\ °C$$

$$T_2 = T_1 - q R_{tKm} = 56,93\ °C$$

$$T_3 = T_2 - q R_{tKe} = 56,81\ °C$$

Ahora para encontrar la temperatura en el centro se usa la ecuación de difusión con generación y se evalúa en el centro.

$$T(r=0) = \frac{\dot{q}\, r_0^2}{4K_0} + T_1 = 99,85\ °C$$

Problema 2

Datos

$K_A = 25\ W/mK$

$K_C = 50\ W/mK$

$L_A = 30 \times 10^{-3}\ m$

$L_B = 30 \times 10^{-3}\ m$

$L_C = 20 \times 10^{-3}\ m$

$T_1 = 261°C$

$T_2 = 211\ °C$

Se pide

$\dot{q}_B = ?$

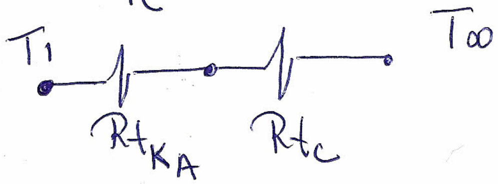
$K_B = ?$

Analogías

(I)

$\rightarrow q_C$

(II)



$\rightarrow q_A$

$$q''_B = q''_A + q''_C \quad \text{(Por unidad de área)}$$

$$R''_{tK_C} = \frac{L_C}{K_C} = 4{,}0 \times 10^{-4}\ \frac{Km^2}{W}$$

$$R''_{tK_A} = \frac{L_A}{K_A} = 1{,}20 \times 10^{-3}\ Km^2/W$$

$$R''_{tc} = \frac{1}{h} = 1 \times 10^{-3}\ Km^2/W$$

$$q''_A = \frac{T_1 - T_\infty}{R''_{tK_A} + R''_{tc}} = 107272{,}73\ \frac{W}{m^2}$$

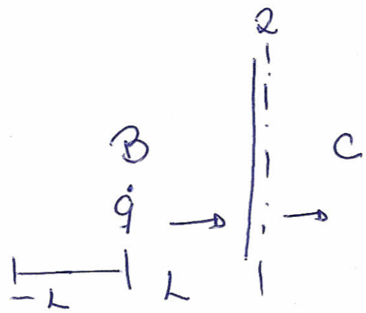
$$q''_C = \frac{T_2 - T_\infty}{R''_{tK_C} + R_{tc}} = 132857{,}14\ \frac{W}{m^2}$$

$$q''_B = 240129{,}87\ W/m^2$$

$$\dot{q}_B = \frac{q''_B}{2L_B} = 4 \times 10^6\ W/m^3$$

⑤

Luego haciendo un balance de energía en la superficie 2



$$q''_{K_B}\Big|_{L_B} = q''_{K_C} = q''_c \text{ (calculado anteriormente)}$$

Luego por ley de Fourier

$$q''_{K_B} = -K_B \frac{dT}{dx}\Big|_{L_B}$$

La ecuación de T(x) se toma para condiciones de borde temperaturas conocidas con generación

$$T(x) = \frac{\dot{q} L_B^2}{2K_B}\left(1 - \frac{x^2}{L_B^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L_B} + \frac{T_1 + T_2}{2}$$

Derivando y evaluando en L_B

$$\frac{dT}{dx}\Big|_{L_B} = -\frac{\dot{q} L_B}{K_B} + \frac{T_2 - T_1}{2L_B} \Rightarrow$$

$$q''_{K_B}\Big|_{L_B} = \dot{q} L_B + \frac{T_1 - T_2}{2L_B} \cdot K_B = q''_c$$

Despejando

$$K_B = \frac{(q''_c - \dot{q} L_B) \cdot 2L_B}{T_1 - T_2} = \frac{(132857{,}40\,W/m^2 - 240129{,}87\,W/m^2) \cdot 2(3\times10^{-2}\,m)}{(261\,°C - 211\,°C)}$$

$$K_B = 15{,}351\ W/m\,°C$$